Lorentz invariant CPT violation: Particle and antiparticle mass splitting

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Abstract

The interpretation of neutrino oscillation data has led to the question whether, in principle, an antiparticle like antineutrino can have a different mass than its particle. In the framework of a Lorentz invariant CPT violation, which is based on the nonlocal interaction vertex and characterized by the infrared divergent form factor, we present an explicit Lagrangian model for the fermion and antifermion mass splitting.

1 Introduction

It is important to study the possible violation of CPT symmetry [1], in particular, in the framework of Lorentz invariant theory. A Lorentz invariant CPT violation, which may be termed as the long distance CPT violation in contrast to the familiar short distance CPT violation [2], has been recently proposed in [3].

The interest in CPT violation and its possible implication on Lorentz invariance breaking has been recently revived due to the neutrino oscillation experiments, whose theoretical interpretation is favoured if the muon antineutrino mass were different than muon neutrino mass [4, 5, 6].

The scheme proposed in [3] is based on the nonlocal interaction vertex and characterized by the infrared divergent form factor. To be definite, the

idea is illustrated by the Yukawa-type Lagrangian

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(x)[i\gamma^\mu\partial_\mu - M]\psi(x) + \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x) \\ &+g\bar{\psi}(x)\psi(x)\phi(x) - V(\phi) \\ &+g_1\bar{\psi}(x)\psi(x)\int d^4y\theta(x^0-y^0)\delta((x-y)^2-l^2)\phi(y).\end{aligned} \tag{1.1}$$

This Lagrangian is formally Hermitian and the term with a small real g_1 and the step function $\theta(x^0-y^0)$ stands for the CPT and T violating interaction; l is a real constant parameter.

We first note that the present way to introduce CPT violation is based on the extra form factor in momentum space as

$$\begin{aligned}&g_1\int d^4x\bar{\psi}(x)\psi(x)\int d^4y\theta(x^0-y^0)\delta((x-y)^2-l^2)\phi(y) \\ &= g_1\int dp_1dp_2dq\int d^4x\bar{\psi}(p_1)e^{-ip_1x}\psi(p_2)e^{-ip_2x} \\ &\times\int d^4y\theta(x^0-y^0)\delta((x-y)^2-l^2)\phi(q)e^{-iqy} \\ &= g_1\int dp_1dp_2dq(2\pi)^4\delta^4(p_1+p_2+q)\bar{\psi}(p_1)\psi(p_2)f(q)\phi(q),\end{aligned} \tag{1.2}$$

where we defined $f(q) \equiv \int d^4z\theta(z^0)\delta(z^2-l^2)e^{iqz}$, namely, CPT violation is realized by an insertion of the form factor $f(q)$ to the $\phi - \psi\bar{\psi}$ coupling in momentum space. The ordinary local field theory is characterized by $\delta(z)$ and $f(q) = 1$. The above form factor is infrared divergent, and it is quadratically divergent in the present example. This infrared divergence arises from the fact that we cannot divide Minkowski space into (time-like) domains with finite *4-dimensional* volumes in a Lorentz invariant manner. The Minkowski space is hyperbolic rather than elliptic. CPT symmetry is related to the fundamental structure of Minkowski space, and thus it is gratifying that its possible breaking is also related to the basic property of Minkowski space.

For the later use, it is convenient to define the form factors

$$f_\pm(p) = \int d^4z_1e^{\pm ipz_1}\theta(z_1^0)\delta((z_1)^2-l^2), \tag{1.3}$$

which are inequivalent for the time-like p due to the factor $\theta(z_1^0)$. For the time-like momentum p, one may choose a suitable Lorentz frame such that

$\vec{p} = 0$ and

$$f_{\pm}(p^0) \;\; = \;\; 2\pi \int_0^{\infty} dz \frac{z^2 e^{\pm i p^0 \sqrt{z^2+l^2}}}{\sqrt{z^2+l^2}}, \tag{1.4}$$

and for the space-like momentum p one may choose a suitable Lorentz frame such that $p^0 = 0$ and

$$f_{\pm}(\vec{p}) \;\; = \;\; \frac{2\pi}{|p|^2} \int_0^{\infty} dz\, z \frac{\sin z}{\sqrt{z^2 + (|p|l)^2}}, \tag{1.5}$$

which is analogous to the Fourier transform of the Coulomb potential and real. The expression $f_{\pm}(p)$ is mathematically related to the formula of the two-point Wightman function (for a free scalar field), which suggests that $f_{\pm}(p)$ is mathematically well-defined for $p \neq 0$ at least in the sense of distribution.

The Lagrangian in (1.1) may be quantized by the path integral by integrating the formal equations of motion by means of Schwinger's action principle [7], whose basis is analogous to that of the Yang–Feldman formulation [8]. We thus have the generating functional $\langle 0, +\infty | 0, -\infty \rangle_J$ with the source term $\mathcal{L}_J = \bar{\psi}(x)\eta(x) + \bar{\eta}(x)\psi(x) + \phi(x)J(x)$, and one may generate Green's functions in a power series expansion of perturbation as

$$(i)^n \langle T^{\star} \phi(x_1)...\phi(x_N) \int d^4y_1 \mathcal{L}_I(y_1) .... \int d^4y_n \mathcal{L}_I(y_n) \rangle, \tag{1.6}$$

where we consider only N scalar particles as external fields, for simplicity. We use the covariant $T^{\star}$-product which is essential to make the path integral on the basis of Schwinger's action principle consistent [7].

On the basis of this quantization, it is confirmed [9] that the time-reversal non-invariance in the square of the probability amplitudes for the processes $\phi \to \bar{\psi}\psi$ and its time reversed formation process,

$$|A(\phi \to \bar{\psi}\psi)|^2 \neq |A(\bar{\psi}\psi \to \phi)|^2, \tag{1.7}$$

is realized after averaging over spin directions for the processes $\phi \to \bar{\psi}\psi$ and $\bar{\psi}\psi \to \phi$ as a result of the interference of two phases θ_i and $\pm\theta_{CPT}$. Here θ_i is the dynamical phase of the Yukawa theory generated by one-loop corrections and $\pm\theta_{CPT}$ is the phase generated by our CPT- and T-violating interaction. This shows that the T-violation in (1.1) is genuine. It is convenient to choose the masses such that $3M > m > 2M$, which makes the above decay mode the only allowed decay mode.

2 Lagrangian model of fermion mass splitting

2.1 Lagrangian formalism

In the present nonlocal formulation, we have a new possibility which is absent in a smooth nonlocal extension of the CPT-even local field theory. The term $i\mu\bar{\psi}(x)\psi(y)$ (to be precise, $i\mu\bar{\psi}(x)\psi(x)$) with a real μ does not appear in the local Lagrangian since it is canceled by its Hermitian conjugate. Also this term is CPT-odd. But in the present nonlocal theory one can consider the Hermitian combination

$$\begin{aligned}&\int d^4x d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)]\\&\quad\times\delta((x-y)^2-l^2)[i\mu\bar{\psi}(x)\psi(y)],\end{aligned}\tag{2.1}$$

which is non-vanishing. Under CPT, we have $i\mu\bar{\psi}(x)\psi(y)\rightarrow -i\mu\bar{\psi}(-y)\psi(-x)$. By performing the change of integration variables $-x\rightarrow y$ and $-y\rightarrow x$, this combination is confirmed to be CPT$=-1$. In fact, we have the following transformation property of the operator part

$$\begin{aligned}&\mathrm{C}:\ i\mu\bar{\psi}(x)\psi(y)\rightarrow i\mu\bar{\psi}(y)\psi(x),\\&\mathrm{P}:\ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})\rightarrow i\mu\bar{\psi}(x^0,-\vec{x})\psi(y^0,-\vec{y}),\\&\mathrm{T}:\ i\mu\bar{\psi}(x^0,\vec{x})\psi(y^0,\vec{y})\rightarrow -i\mu\bar{\psi}(-x^0,\vec{x})\psi(-y^0,\vec{y}),\end{aligned}\tag{2.2}$$

and thus the overall transformation property is C$=-1$, P$=1$, T$=1$. Namely, C$=$CP$=$CPT$=-1$.

It is thus interesting to examine a new action

$$\begin{aligned}S &= \int d^4x\{\bar{\psi}(x)i\gamma^\mu\partial_\mu\psi(x)-m\bar{\psi}(x)\psi(x)\\&\quad-\int d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)]\delta((x-y)^2-l^2)\\&\quad\times[i\mu\bar{\psi}(x)\psi(y)]\},\end{aligned}\tag{2.3}$$

which is Lorentz invariant and Hermitian. For the real parameter μ, the third term has C$=$CP$=$CPT$=-1$ and no symmetry to ensure the equality of particle and antiparticle masses.

The Dirac equation is replaced by

$$\begin{aligned}&i\gamma^\mu\partial_\mu\psi(x)=m\psi(x)\\&+i\mu\int d^4y[\theta(x^0-y^0)-\theta(y^0-x^0)]\delta((x-y)^2-l^2)\psi(y).\end{aligned}\tag{2.4}$$

By inserting an ansatz for the possible solution

$$\psi(x) = e^{-ipx}U(p), \tag{2.5}$$

we have

$$\begin{aligned}
\not{p}U(p) &= mU(p) \\
&+ i\mu \int d^4y[\theta(x^0 - y^0) - \theta(y^0 - x^0)] \\
&\times \delta((x-y)^2 - l^2)e^{-ip(y-x)}U(p) \\
&= mU(p) + i\mu[f_+(p) - f_-(p)]U(p),
\end{aligned} \tag{2.6}$$

where $f_\pm(p)$ is the Lorentz invariant form factor defined in (1.3).

The (off-shell) propagator is defined by

$$\begin{aligned}
&\int d^4x e^{ip(x-y)} \langle T^\star \psi(x)\bar{\psi}(y)\rangle \\
&= \frac{i}{\not{p} - m + i\epsilon - i\mu[f_+(p) - f_-(p)]},
\end{aligned} \tag{2.7}$$

which is manifestly Lorentz covariant. Note that we use the $T^\star$-product for the path integral in accord with Schwinger's action principle, which is based on the equation of motion (2.4) with a source term added:

$$\langle 0, +\infty | 0, -\infty\rangle_J = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp i\{S + \int d^4x \mathcal{L}_J]\}, \tag{2.8}$$

where the action S is given in (2.3) and the source term is $\mathcal{L}_J = \bar{\psi}(x)\eta(x) + \bar{\eta}(x)\psi(x)$. The $T^\star$-product is quite different from the canonical T-product in the present nonlocal theory, and in fact the canonical quantization is not defined in the present theory. It is however important to note that the $T^\star$-product can reproduce all the results of the T-product, if the T-product is well-defined, by means of the Bjorken–Johnson–Low prescription [7]. In the present example, the presence of the sine-function in the denominator of the correlation function complicates this procedure, which is an indication of the absence of the canonical quantization of (2.3). We also emphasize that the analysis of the mass-splitting can be performed in terms of the exact solution of the (modified) free Dirac equation (2.4), which also defines the propagator in the present path integral prescription. After all, Dirac discovered the antiparticle by solving his equation exactly. The propagator (2.7) is also an exact propagator for (2.3) in the sense of the propagator

theory of relativistic quantum mechanics, and thus it could describe the particle and antiparticle propagation if one understands the antiparticle with negative energy propagating backward in time. However, if one attempts to describe the particle and antiparticle propagation with definite masses by pole approximation, for example, then the off-shell Lorentz covariance of the propagator (2.7) is lost, as is discussed later (see eq. (2.17)).

For the space-like p, the extra term with μ in the denominator of the propagator (2.7) vanishes since $f_+(p) = f_-(p)$ for $p = (0, \vec{p})$, as is shown in (1.5). Thus the propagator has poles only at the time-like momentum, and in this sense the present Hermitian action (2.3) does not allow a tachyon. By assuming a time-like p, we go to the frame where $\vec{p} = 0$. Then the eigenvalue equation is given by

$$p_0 = \gamma_0\{m + i\mu[f_+(p_0) - f_-(p_0)]\}, \tag{2.9}$$

namely,

$$p_0 = \gamma_0\left[m - 4\pi\mu\int_0^\infty dz \frac{z^2\sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}\right], \tag{2.10}$$

where we used the explicit formula in (1.4). The solution p_0 of this equation (2.10) determines the possible mass eigenvalues.

This eigenvalue equation under $p_0 \to -p_0$ becomes:

$$-p_0 = \gamma_0\left[m + 4\pi\mu\int_0^\infty dz \frac{z^2\sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}\right]. \tag{2.11}$$

By sandwiching this equation by γ_5, which is regarded as CPT operation, we have

$$-p_0 = \gamma_0\left[-m - 4\pi\mu\int_0^\infty dz \frac{z^2\sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}\right], \tag{2.12}$$

i.e.,

$$p_0 = \gamma_0\left[m + 4\pi\mu\int_0^\infty dz \frac{z^2\sin[p_0\sqrt{z^2+l^2}]}{\sqrt{z^2+l^2}}\right], \tag{2.13}$$

which is not identical to the original equation in (2.10). In other words, if p_0 is the solution of the original equation, $-p_0$ cannot be the solution of the

original equation except for $\mu = 0$. The last term in the Lagrangian (2.3) with C=CP=CPT=-1 splits the particle and antiparticle masses.

As a crude estimate of the mass splitting, one may assume $\mu \ll m$ and solve these equations iteratively. If the particle mass for (2.10) is chosen at

$$p_0 \simeq m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}}, \tag{2.14}$$

then the antiparticle mass for (2.13) is estimated at

$$p_0 \simeq m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m\sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}}. \tag{2.15}$$

2.2 Canonical description

Once one finds eigenvalues, one may examine the behavior of the off-shell propagator (2.7) around those pole positions approximately and may apply the Bjorken–Johnson–Low (BJL) prescription to reveal the canonical structure [7]. Then one finds an operator description of those particle and antiparticle with different masses, although the manifest invariance is lost. (This is somewhat analogous to the electromagnetic field. The off-shell Maxwell equation is manifestly Lorentz covariant but if one applies the physical Coulomb gauge to define the photon, the manifest invariance is lost.) We would like to explain the basic steps of this procedure.

If one denotes the particle mass by m_+ and antiparticle mass by m_-, respectively, we have *approximately* near the pole positions of the propagator in (2.7):

$$\begin{aligned} \int d^4x e^{ip(x-y)} \langle T^\star \psi(x)\bar{\psi}(y)\rangle \quad &\simeq \quad \frac{i}{\not{p} - m_+ + i\epsilon}, \text{ for } p_0 > 0, \\ &\simeq \quad \frac{i}{\not{p} - m_- + i\epsilon}, \text{ for } p_0 < 0. \end{aligned} \tag{2.16}$$

The first step of BJL prescription is to examine the large p_0 behavior of the right-hand side of the Fourier transform, which goes to 0 in the present case. In this case, we replace the $T^\star$-product by the canonical T-product.

From the point of view of the field-product $\psi(x)\bar{\psi}(y)$, T is supposed to specify the product even for the precise coincident time $x^0 = y^0$, while $T^\star$ specifies the product only for $x^0 \neq y^0$ and examine the behavior of the product for $x^0 - y^0 \to 0$ later. These two procedures agree with each other

for the theory where ordinary canonical quantization is well-defined, but in general they do not agree with each other. The Schwinger term, for example, is identified by this disagreement. If the short-time limit is well-specified by T, the Riemann-Lebesgue lemma in the Fourier transform implies that the large frequency limit of the T-product vanishes. This is the basis of the above replacement of $T^\star$ by T [7].

We thus have the relations, which are more specific than (2.16) but still approximate, although we use the equality symbol:

$$
\begin{aligned}
\int d^4x e^{ip(x-y)}\langle T\psi_+(x)\bar{\psi}_+(y)\rangle &= \frac{i\Lambda_+(m_+)}{p^2-m_+^2+i\epsilon}, \\
&\quad \text{for} \ \ p_0>0, \\
\int d^4x e^{ip(x-y)}\langle T\psi_-(x)\bar{\psi}_-(y)\rangle &= \frac{i\Lambda_-(m_-)}{p^2-m_-^2+i\epsilon}, \\
&\quad \text{for} \ \ p_0<0,
\end{aligned}
\tag{2.17}
$$

where we have separated ψ into positive $\psi_+(x)$ and negative $\psi_-(x)$ frequency components. We used the positive energy and negative energy projection operators constructed by the solutions of (2.6), where $\Lambda_+(m)+\Lambda_-(m)=\not{p}+m$ for the equal mass case. The final step of BJL prescription is to multiply both-hand sides of the relations in (2.17) by p_0 and consider the large p_0 limit. For example,

$$
\begin{aligned}
& p_0\int d^4x e^{ip(x-y)}\langle T\psi_+(x)\bar{\psi}_+(y)\rangle \\
= & -i\int d^4x\frac{\partial}{\partial x^0}e^{ip(x-y)}\langle T\psi_+(x)\bar{\psi}_+(y)\rangle \\
= & \ i\int d^4x e^{ip(x-y)}\frac{\partial}{\partial x^0}\langle T\psi_+(x)\bar{\psi}_+(y)\rangle \\
= & \ i\int d^4x e^{ip(x-y)}[\langle\delta(x^0-y^0)\{\psi_+(x),\bar{\psi}_+(y)\}\rangle \\
+ & \ \langle T\frac{\partial}{\partial x^0}\psi_+(x)\bar{\psi}_+(y)\rangle] = \frac{ip_0\Lambda_+(m_+)}{p^2-m_+^2+i\epsilon}.
\end{aligned}
\tag{2.18}
$$

We consider the limit $p_0\to\infty$ in this relation. The T-product part $\langle T\frac{\partial}{\partial x^0}\psi_+(x)\bar{\psi}_+(y)\rangle$ goes to 0 in this limit since it is always *defined* to satisfy the Riemann-Lebesgue type condition, which specifies the separation between the commutator part and the T-product part uniquely. We thus conclude from the

last two relations in (2.18)

$$\delta(x^0 - y^0)\{\psi_+(x), \psi_+^\dagger(y)\} = \frac{1}{2}\delta^4(x-y),$$
$$\delta(x^0 - y^0)\{\psi_-(x), \psi_-^\dagger(y)\} = \frac{1}{2}\delta^4(x-y), \tag{2.19}$$

where the second relation follows from the second relation in (2.17). Note that at extremely high energies, the mass difference does not matter at least in the fixed mass approximation. By this way, we obtain an approximate canonical description of the CPT-violated fermion with mass splitting. The basic approximation involved in the transition from the manifestly Lorentz covariant off-shell propagator to the approximate canonical description is the identification of the pole structure in (2.16), which is exact for the case of an identical (momentum-independent) mass.

3 Conclusion

We have presented a simple Lorentz invariant CPT violating Lagrangian model in (2.3), which produces the splitting of particle and antiparticle masses. The simple Lagrangian model will provide a useful theoretical laboratory when one investigates Lorentz invariant CPT violation effects.

Besides the fact that both CPT and Lorentz invariance are two most fundamental symmetries in physics, whose violations have not been hitherto observed, the relation between the two symmetries and their possible breaking are of considerable theoretical and experimental interest. Recent MINOS neutrino experiments with their favoured interpretation through a mass difference for muon neutrino and antineutrino have revived interest in CPT violation and its possible implication on Lorentz invariance breaking [4, 5, 6].

It is an interesting question whether the CPT violation in our model could be a long distance effective description of some modified structure of space-time at short distances, for example.

Our Lagrangian is nonlocal and the local gauge principle cannot be directly applied to it. Nevertheless, its novelty is the specific realization of a CPT-odd and Lorentz invariant model, showing a mass splitting of fermions and, in general, of any particle and its antiparticle.

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References

[1] W. Pauli, *Niels Bohr and the Development of Physics*, W. Pauli (ed.), Pergamon Press, New York, 1955; G. Lüders, Det. Kong. Danske Videnskabernes Selskab, Mat.-fys. Medd. **28** (5) (1954).

[2] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. A **11**, 1489 (1996); J. R. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D **53**, 3846 (1996); N. E. Mavromatos, J. Phys. Conf. Ser. **171**, 012007 (2009).

[3] M. Chaichian, A.D. Dolgov, V.A. Novikov and A. Tureanu, Phys. Lett. B**699**, 177 (2011).

[4] H. Murayama and T. Yanagida, Phys. Lett. B **520**, 263 (2001); G. Barenboim, L. Borissov, J. D. Lykken and A. Y. Smirnov, JHEP **0210**, 001 (2002); G. Barenboim, L. Borissov and J. Lykken, Phys. Lett. B **534**, 106 (2002); S. M. Bilenky, M. Freund, M. Lindner, T. Ohlsson and W. Winter, Phys. Rev. D **65**, 073024 (2002); G. Barenboim and J. D. Lykken, Phys. Rev. D **80**, 113008 (2009).

[5] G. Altarelli, "The Mystery of Neutrino Mixings", arXiv:1111.6421 [hep-ph].

[6] P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. Lett. **107**, 181802 (2011); see, however, P. Adamson *et al.* [MINOS Collaboration], *An improved measurement of muon antineutrino disappearance in MINOS* [arXiv:1202.2772 [hep-ex]]

[7] K. Fujikawa, Phys. Rev. D**70**, 085006 (2004).

[8] C.N. Yang and D. Feldman, Phys. Rev. **78**, 972 (1950).

[9] M. Chaichian, K. Fujikawa and A. Tureanu, *Lorentz invariant CPT violation*, in preparation.